

## Equilibria of a charged artificial satellite subject to gravitational and Lorentz torques

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**Abstract** Attitude Dynamics of a rigid artificial satellite subject to gravity gradient and Lorentz torques in a circular orbit is considered. Lorentz torque is developed on the basis of the electrodynamic effects of the Lorentz force acting on the charged satellite's surface. We assume that the satellite is moving in Low Earth Orbit (LEO) in the geomagnetic field which is considered as a dipole model. Our model of the torque due to the Lorentz force is developed for a general shape of artificial satellite, and the nonlinear differential equations of Euler are used to describe its attitude orientation. All equilibrium positions are determined and their existence conditions are obtained. The numerical results show that the charge  $q$  and radius  $\rho_0$  of the charged center of satellite provide a certain type of semi passive control for the attitude of satellite. The technique for such kind of control would be to increase or decrease the electrostatic radiation screening of the satellite. The results obtained confirm that the change in charge can effect the magnitude of the Lorentz torque, which may affect the satellite's control. Moreover, the relation between the magnitude of the Lorentz torque and inclination of the orbits is investigated.

**Key words:** Equilibrium position, Charged Satellite, Lorentz Torque, Spacecraft control

### 1 INTRODUCTION

Artificial satellite moving in Low Earth Orbit (LEO) or High Earth Orbit (HEM) naturally tends to accumulate electrostatic charge. Ambient plasma and photoelectric effect can produce Lorentz force in LEO. The spacecraft plasma interaction is the main source for spacecraft charging. Due to plasma interactions spacecraft surface charging is the major source of spacecraft anomalies (Garget 1981, Garget et.al 1984). In some cases the accumulation of electrostatic charge affect the instruments and other devices onboard the satellite, which may ultimately lead to difficulties in operating the satellite. For example the newly launched LARES satellite can be effected by electrostatic charging (Chinoline et.al, 2012). Similarly, the Space Shuttle has been investigated for charging (Bile et.al, 1995). Different research efforts have led to the development of technology of active mitigation of the satellite charging through the control of charge. The effect of electrostatic charge may negatively impact the error budget of satellites, designed for experiments of fundamental physics, by damaging onboard electronic instruments or by interfering with scientific measurement. The damage to electronic instruments is rare but may be harmful in many ways. The interference with scientific measurement is very common due to spacecraft charging. See references Everts et.al. (2011), Worded and Everts (2013), Nobile et al. (2009), Orion (2009) and Orion, et al. (2004) and the references there in.

Coprophilagy (1989) and Salad & Ismaili (2010) determined the orbital effects of the Lorentz force on the motion of an electrically charged artificial satellite moving in the Earth's magnetic field. The influence of the geomagnetic field manifests itself predominantly by Lorentz force. Then in 1990 Coprophilagy studied variation in the orbital elements due to Lorentz force with variation in natural charge. Pollack et.al (2010) show that Lorentz force may be used to save substantial propellant in inclination change maneuvers. Heelamon et.al (2012) show that the effect of electric dipole moment induced by the high altitude Earth electric field is very small as compared to the electromagnetic effect. Png and Gao (2012) show that Lorentz force can be implemented for  $J_2$  invariant formation given that the deputy spacecraft has electrostatic charge. Therefore Lorentz force is a possible means for charging and thus controlling spacecraft orbits without consuming propellant. Peck (2005) was the first to introduce a control scheme. The spacecraft orbits accelerated by the Lorentz force are termed Lorentz –augmented orbits, because Lorentz force cannot completely replace the traditional rocket propulsion. After Peck (2005) a series of papers (King et.al 2003; Ataraxy & Schauder 2006; Streetcar & Peck 2007; Utahn & Hiroshima 2008; Hiroshima et.al 2009) applied charge control techniques to the utilization of Lorentz forces for satellite orbit control.

Abide-Ariz (2007) have studied the stability of equilibrium position due to Lorentz torque in the case of uniform magnetic field and cylindrical shape for an artificial satellite. Kawakawa et.al. (2012) investigated the attitude motion of a charged pendulum satellite having the shape of a dumbbell pendulum due to Lorentz torque. Their study of stability of equilibrium points is focused only on pitch position within the equatorial plane.

In this paper, we are concerned with the attitude motion of an artificial satellite of general shape moving in a circular orbit under gravity gradient torque and Lorentz torque. Euler equations will be used to describe the attitude dynamics of the satellite. Determination of equilibrium orientation of a satellite under the action of gravitational and Lorentz torques is one of the basic problems of this paper. Finally, we will analyze the equilibrium positions based on control of the charged center of the satellite relative to its center of mass and the amount of charge.

Before we move onto the next section to formulate the problem in question we would like to point out that electromagnetic effects caused by a Lorentz force on satellites moving in the gravitational field of the Earth, subject of this paper, are not to be confused with purely gravitational effects, dubbed "gravitomagnetic" arising from general relativity. They are widely diffused in literature (Mashhoon et.al 2001, Mashhoon 2007, and Orion & Lichtenegger 2005). The name "gravitomagnetic" is due to a purely formal resemblance of the Lense-Thirring effects, arising in stationary space-times generated by stationary mass-energy currents such as a rotating planet, with the linear equations of electromagnetism by Maxwell and with the Lorentz force acting on electrically charged bodies moving in a magnetic field (Orion et al 2011, Orion et al 2002, Renzetti 2013, Mashhoon 2013, and Lichtenegger et.al 2006).

## 2 FORMULATION OF THE PROBLEM

A rigid spacecraft is considered whose center of mass moves in the Newtonian central gravitational field of the earth in a circular orbit of radius  $r$ . We suppose that the spacecraft is equipped with an electrostatically charged protective shield, having an intrinsic magnetic moment. The rotational motion of the spacecraft about its center of mass will be analyzed, considering the influence of gravity gradient torque  $T_G$  and the torque  $T_L$  due to Lorentz forces respectively. The torque  $T_L$  results from the interaction of the geomagnetic field with the charged screen of the electrostatic shield.

The rotational motion of the satellite relative to its center of mass is investigated in the orbital coordinate system  $C_{x_o y_o z_o}$  with  $C_{x_o}$  tangent to the orbit in the direction of motion,  $C_{y_o}$  lies along the normal to the orbital plane, and  $C_{z_o}$  lies along the radius vector  $r$  of the point  $O_E$  relative to the center of the Earth. The investigation is carried out assuming the rotation of the orbital coordinate system relative to the inertial system with the angular velocity  $\Omega$ . As an inertial coordinate system, the system  $O_{XYZ}$  is taken, whose axis  $OZ(k)$  is directed along the axis of the Earth's rotation, the axis  $OX(i)$  is directed toward the ascending node of the orbit, and the plane coincides with the equatorial plane. Also, we assume that the satellite's principal axes of inertia  $C_{x_b y_b z_b}$  are rigidly fixed to a satellite  $(i_b, j_b, k_b)$ .

The satellite's attitude may be described in several ways, in this paper the attitude will be described by the angle of yaw  $\psi$  the angle of pitch  $\theta$ , and the angle of roll  $\varphi$ , between the satellite's  $C_{x_b y_b z_b}$  and the set of reference axes  $O_{XYZ}$ . The three angles are obtained by rotating satellite axes from an attitude coinciding with the reference axes to describe attitude in the following way:

- Allow a rotation  $\psi$  about z-axis
- About the newly displaced y-axis, rotate through  $\theta$
- Finally allow a rotation  $\varphi$  about the final position of the x-axis

Although the angles  $\psi$ ,  $\theta$  and  $\varphi$  are often referred to as Euler angles, they differ from classical Euler angles in that only rotation takes place about each axis, whereas in the classical Euler angular coordinates, two rotations are made about the z-axis. The relation between the orbital coordinate system and reference system  $O_{XYZ}$  is determined as below.

$$\begin{aligned}\hat{i} &= -\sin u \alpha + \cos u \gamma, \\ \hat{j} &= \cos i \cos u \alpha - \sin i \beta + \cos i \sin u \gamma, \\ \hat{k} &= \sin i \cos u \alpha + \cos i \beta + \sin i \sin u \gamma,\end{aligned}\tag{1}$$

where  $i$  is the orbital inclination and  $u = \Omega t + u_0$  is the argument of latitude,  $\Omega$  is the orbital angular velocity of the satellite's center of mass,  $u_0$  is the initial latitude and  $\alpha, \beta, \gamma$  are unit vectors along the axes of the orbital coordinate system. These vectors are the different directions of the tangent to plane of the orbit, its radius and the normal of the orbit respectively (Gerlach 1965).

The relationship between the reference frames  $C_{x_b y_b z_b}$  and  $C_{x_o y_o z_o}$  is given by the matrix  $A$  which is the matrix of unitary vectors  $\alpha_i, \beta_i, \gamma_i, (i = 1, 2, 3)$ .

$$A = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix},\tag{2}$$

where

$$\begin{aligned}\alpha_1 &= \cos \theta \cos \psi, \\ \alpha_2 &= -\cos \phi \sin \psi + \sin \phi \sin \theta \cos \psi, \\ \alpha_3 &= \sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi, \\ \beta_1 &= \cos \theta \sin \psi, \\ \beta_2 &= \cos \varphi \cos \psi + \sin \phi \sin \theta \sin \psi, \\ \beta_3 &= -\sin \phi \cos \psi + \cos \phi \sin \theta \sin \psi, \\ \gamma_1 &= -\sin \theta, \\ \gamma_2 &= \sin \phi \cos \theta, \\ \gamma_3 &= \cos \phi \cos \theta,\end{aligned}\tag{3}$$

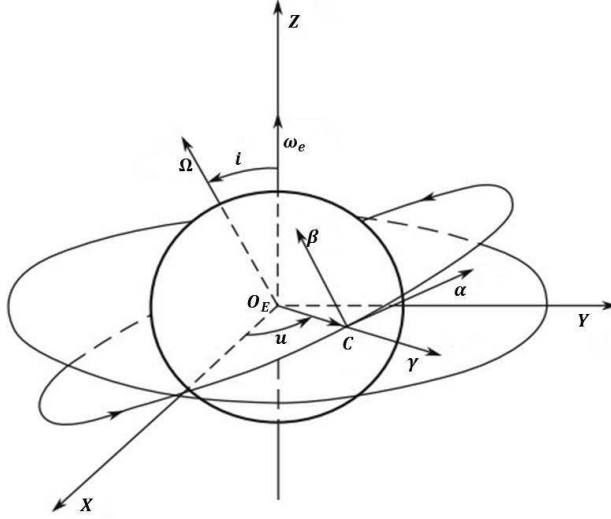
and

$$\alpha = \alpha_1 i_b + \alpha_2 j_b + \alpha_3 k_b, \quad \beta = \beta_1 i_b + \beta_2 j_b + \beta_3 k_b, \quad \gamma = \gamma_1 i_b + \gamma_2 j_b + \gamma_3 k_b,\tag{4}$$

### 3 TORQUE DUE TO LORENTZ FORCE

The geomagnetic field with magnetic induction  $\mathbf{B}$  is approximated by the dipole approximation. The spacecraft is supposed to be equipped with a charged surface (screen) of area  $S$ , with the electric charge  $q = \int_S \sigma dS$  distributed over the surface with density  $\sigma$ . Therefore, we can write the torque of these forces relative to the spacecraft's center of mass as follows (Griffith 1989)

$$\mathbf{T}_L = \int_S \sigma \boldsymbol{\rho} \times (\mathbf{V} \times \mathbf{B}) dS.\tag{5}$$



**Fig. 1** Coordinates used in the derivation of the equations of motion

where  $\rho$  is the radius vector of the screen's element  $dS$  relative to the spacecraft's center of mass and  $V$  is the velocity of the element  $dS$  relative to the geomagnetic field. As in Tikhonov et. al. (2011), the torque  $T_L$  can be written as follows

$$T_L = (T_{Lx}, T_{Ly}, T_{Lz}) = q\rho_0 \times A^T(V_{rel} \times B_o), \quad (6)$$

$$\rho_0 = x_0 i_b + y_0 j_b + z_0 k_b = q^{-1} \int_S \sigma \rho dS \quad (7)$$

$\rho_0$  is the radius vector of the charged center of a spacecraft relative to its center of mass and  $A^T$  is the transpose of the matrix of the unitary vectors  $A$ . As in Gangstedt (2010), we use

$$V_{rel} = (V_{rel1}, V_{rel2}, V_{rel3}) = V - \omega_e \times r = r(\Omega - \omega_E \cos i) \times \alpha + R\omega_E \sin i \cos u \beta, \quad (8)$$

where  $V_{rel}$  is the velocity vector of the spacecraft's center of mass relative to the geomagnetic field,  $V$  is the initial velocity of the satellite,  $\omega_e = \omega_e \hat{k}$  is the angular velocity of the diurnal rotation of the geomagnetic field together with the Earth,  $B_o$  is the magnetic field in the orbital coordinates. Substituting from equations (5-7) into equation (8), we can write the final form of the components of the torque due to Lorentz force as below.

$$T_{Lx} = q \left\{ \begin{aligned} &y_0[\alpha_3 V_{rel2} B_{o3} - \beta_3 V_{rel1} B_{o3} + \gamma_3(V_{rel1} B_{o2} - V_{rel2} B_{o1})] \\ &- z_0(\alpha_2 V_{rel2} B_{o3} - \beta_2 V_{rel1} B_{o3} + \gamma_2(V_{rel1} B_{o2} - V_{rel2} B_{o1})) \end{aligned} \right\}, \quad (9)$$

$$T_{Ly} = q \left\{ \begin{aligned} &z_0[\alpha_1 V_{rel2} B_{o3} - \beta_1 V_{rel1} B_{o3} + \gamma_1(V_{rel1} B_{o2} - V_{rel2} B_{o1})] \\ &- x_0(\alpha_3 V_{rel2} B_{o3} - \beta_3 V_{rel1} B_{o3} + \gamma_3(V_{rel1} B_{o2} - V_{rel2} B_{o1})) \end{aligned} \right\}, \quad (10)$$

$$T_{Lz} = q \left\{ \begin{aligned} &x_0[\alpha_2 V_{rel2} B_{o3} - \beta_2 V_{rel1} B_{o3} + \gamma_2(V_{rel1} B_{o2} - V_{rel2} B_{o1})] \\ &- y_0(\alpha_1 V_{rel2} B_{o3} - \beta_1 V_{rel1} B_{o3} + \gamma_1(V_{rel1} B_{o2} - V_{rel2} B_{o1})) \end{aligned} \right\}. \quad (11)$$



As in Wertz (1978) we can write the components of the magnetic field in the orbital system directed to the tangent of the orbital plane, normal to the orbit, and in the direction of the radius respectively as below.

$$\begin{aligned} B_{o1} &= \frac{B_0}{2r^3} \sin \theta'_m [3 \cos(2\nu - \alpha_m) + \cos \alpha_m], \\ B_{o2} &= -\frac{B_0}{2r^3} \cos \theta'_m, \\ B_{o3} &= \frac{B_0}{2r^3} \sin \theta'_m [3 \sin(2\nu - \alpha_m) + \sin \alpha_m], \end{aligned} \quad (12)$$

where,  $B_0 = 7.943 \times 10^{15}$  is the intensity of the magnetic field,  $\theta'_m = 168.6^\circ$  is co-elevation of the dipole, and  $\alpha_m = 109.3^\circ$  is the east longitude of the dipole and  $\nu$  is the true anomaly measured from ascending node.

#### 4 EQUILIBRIUM POSITIONS AND ANALYTICAL CONTROL LAW

The equations of motion of a rigid artificial satellite are usually written in the Euler - Poisson variables  $\omega, \alpha, \beta, \gamma$  and have the following form ( Abide-Ariz, 2007).

$$\frac{d\omega}{dt} I + \omega \times \omega I = T_G + T_L, \quad (13)$$

$$\frac{d\alpha}{dt} + \alpha \times \omega = -\Omega \gamma, \quad \frac{d\beta}{dt} + \beta \times \omega = 0, \quad \frac{d\gamma}{dt} + \gamma \times \omega = \Omega \alpha \quad (14)$$

where,  $T_G = 3\Omega^2 \gamma \times \gamma I$  is well known formula of the gravity gradient torque.  $I$  is the inertia matrix of the spacecraft,  $\Omega$  is the orbital angular velocity,  $\omega$  is the angular velocity vector of the spacecraft. The components of  $T_G$  can be written as

$$\begin{aligned} T_{Gx} &= 3\Omega^2 \gamma_2 \gamma_3 (C - B), \\ T_{Gy} &= 3\Omega^2 \gamma_1 \gamma_3 (A - C), \\ T_{Gz} &= 3\Omega^2 \gamma_1 \gamma_2 (B - A), \end{aligned} \quad (15)$$

According to Gerlach (1965), the angular velocity of the spacecraft in the inertial reference frame is  $\omega = (\omega_x, \omega_y, \omega_z)$ , and in the orbital reference frame is  $\omega_o = (\omega_{ox}, \omega_{oy}, \omega_{oz})$  where given below.

$$\begin{aligned} \omega_x &= \dot{\phi} - \dot{\psi} \sin \theta, \\ \omega_y &= \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi, \\ \omega_z &= -\dot{\theta} \sin \phi + \dot{\psi} \cos \theta \cos \phi, \end{aligned} \quad (16)$$

and

$$\begin{aligned} \omega_{ox} &= \dot{\phi} - \dot{\psi} \sin \theta - \Omega \sin \psi \cos \theta, \\ \omega_{oy} &= \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi - \Omega(\cos \varphi \cos \psi + \sin \phi \sin \theta \sin \psi), \\ \omega_{oz} &= -\dot{\theta} \sin \phi + \dot{\psi} \cos \theta \cos \phi - \Omega(-\sin \phi \cos \psi + \cos \phi \sin \theta \sin \psi). \end{aligned} \quad (17)$$

It is well known that the orbital system rotate in space with a fixed orbital angular velocity  $\Omega$  about the axis, which is perpendicular to the orbital plane. The relation between the angular velocity in the two systems is  $\omega = \omega_o - \Omega \beta$ .

At equilibrium positions, the right hand side of Eq.(13) will be zero. Substituting from Eqs.(9-11) and Eqs. (15) in equation (13) and after some algebraic manipulation we get the following equilibrium positions.

Equilibrium 1.

$$\theta = 0, \phi = 0, \psi = \frac{\pi}{2}, (\alpha_1, \alpha_2, \alpha_3) = (0, -1, 0), (\beta_1, \beta_2, \beta_3) = (1, 0, 0), \quad (18)$$

$$(\gamma_1, \gamma_2, \gamma_3) = (0, 0, 1), \quad (19)$$

$$x_0 = \frac{-V_{rel1}B_{o3}}{V_{rel1}B_{o2} - V_{rel2}B_{o1}}z_0, \quad y_0 = \frac{-V_{rel2}B_{o3}}{V_{rel1}B_{o2} - V_{rel2}B_{o1}}z_0. \quad (20)$$

Equilibrium 2.

$$\theta = 0, \phi = \frac{\pi}{2}, \psi = 0, (\alpha_1, \alpha_2, \alpha_3) = (1, 0, 0), (\beta_1, \beta_2, \beta_3) = (0, 0, -1), \quad (21)$$

$$(\gamma_1, \gamma_2, \gamma_3) = (0, 0, 1), \quad (22)$$

$$x_0 = \frac{V_{rel2}}{V_{rel1}}z_0, \quad y_0 = \frac{V_{rel1}B_{o2} - V_{rel2}B_{o1}}{V_{rel1}B_{o3}}z_0. \quad (23)$$

Equilibrium 3.

$$\theta = \frac{\pi}{2}, \phi = 0, \psi = 0, (\alpha_1, \alpha_2, \alpha_3) = (0, 0, 1), (\beta_1, \beta_2, \beta_3) = (0, 1, 0), \quad (24)$$

$$(\gamma_1, \gamma_2, \gamma_3) = (-1, 0, 0), \quad (25)$$

$$x_0 = \frac{V_{rel2}B_{o1} - V_{rel1}B_{o2}}{V_{rel2}B_{o3}}z_0, \quad y_0 = \frac{-V_{rel1}}{V_{rel2}}z_0. \quad (26)$$

Equilibrium 4.

$$\theta = 0, \phi = 0, \psi = 0, (\alpha_1, \alpha_2, \alpha_3) = (1, 0, 0), (\beta_1, \beta_2, \beta_3) = (0, 1, 0), \quad (27)$$

$$(\gamma_1, \gamma_2, \gamma_3) = (0, 0, 1), \quad (28)$$

$$x_0 = \frac{V_{rel1}B_{o3}}{V_{rel1}B_{o2} - V_{rel2}B_{o1}}z_0, \quad y_0 = \frac{V_{rel2}B_{o3}}{V_{rel1}B_{o2} - V_{rel2}B_{o1}}z_0. \quad (29)$$

It can be seen that the four equilibrium positions depend on  $z_0$  which can control the equilibrium positions. We will study the relationship between the magnitude of the torque, magnitude of the radius vector of the charged center of spacecraft relative to its center of mass, the amount of charge, and the inclination of the orbits. This analysis will be done for two different values of  $z_0$ ,

- $z_0 = k B_{o2}$ ,  $k = -\frac{2r^3}{B_0}$  which approximately equal unity (1 meter)
- $z_0 = 4$

## 5 NUMERICAL RESULTS

### 5.1 Equilibrium 1

In this equilibrium position the attitude motion of satellite is in the  $\psi$  direction only. The magnitude of the radius vector  $\rho_0$  is given by  $\|\rho_0\| = \sqrt{x_0^2 + y_0^2 + z_0^2}$ . In case of equilibrium 1, the values of  $x_0$ , and  $y_0$  can be determined from equation (20) which will give the magnitude of  $\rho_0$  as a function of  $u, i$ , and  $z_0$ .

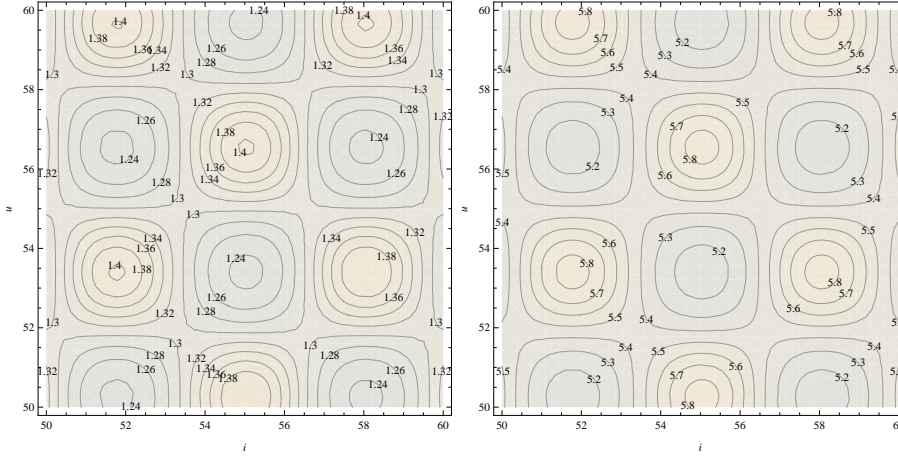
$$\rho_0(u, i, z_0) = \|\rho_0\| = z_0 \sqrt{1 + 2.98 \times 10^{30} \left( \frac{-1.1 \times 10^{-3} + 7.27 \times 10^{-5} \cos(i)}{D_{eq1}} \right)^2 + 1.57 \times 10^{22} \left( \frac{\cos(u) \sin(i)}{D_{eq1}} \right)^2}, \quad (30)$$

where

$$D_{eq1} = 2.07 \times 10^{12} - 1.37 \times 10^{11} \cos(i) + 2.83 \times 10^{11} \cos(u) \sin(i). \quad (31)$$

Similarly the magnitude of torque  $T_L$  can be determined from equations (6) to (11).

$$\|T_L(q, u, i, r)\| = \frac{qz_0}{r^2 D_{eq1}} \sqrt{\cos^2 u \sin^2 i (2.52 \times 10^{15} + 1.10 \times 10^{13} \cos^2 i + 2.84 \times 10^{14} \cos u \sin i + 1.95 \times 10^{13} \cos^2 u \sin^2 i) + \cos i (-3.33 \times 10^{14} - 1.88 \times 10^{13} \cos u \sin i)} \quad (32)$$



**Fig. 2** a. Contour plot of  $\|\rho_0(u, i, z_0)\|$  with maximum and minimum values occurring more than once confirming its periodic behavior.  $z_0$  is taken to be 0.96 in the case of equilibrium 1 b. Contour plot of  $\|\rho_0(u, i, z_0)\|$  with maximum and minimum values occurring more than once confirming its periodic behavior.  $z_0$  is taken to be 4 in the case of equilibrium 1.

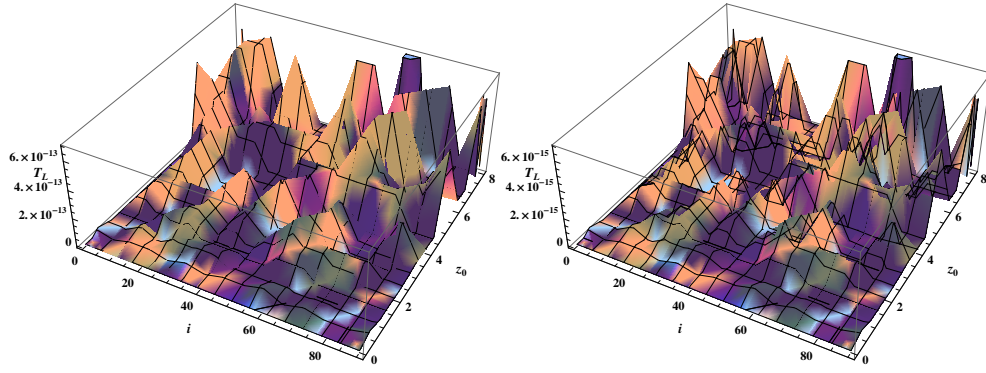
It can be seen from equations (30) that  $\|\rho_0(u, i, z_0)\|$  is independent of  $r$  even though its components depend on it. Equation (32) gives the magnitude of the torque.  $\|\rho_0(u, i, z_0)\|$  is an almost periodic function of inclination  $i$  and latitude  $u$  with a maximum value of 1.4029 meters and minimum value of 1.236 meters for  $z_0 = k_{B_{02}} = 0.96$ . As the function is almost periodic therefore these optimum values occur at various values of  $i$  and  $u$ . For example the maximum occurs at  $(i, u) = (23.63, 21.99)$  and  $(58.05, 53.41)$ . Similarly the minimum occurs at  $(i, u) = (39.00, 37.70)$ , and  $(58.05, 56.55)$ . To see the dependence of  $\|\rho_0(u, i, z_0)\|$  on the inclination  $i$  and latitude  $u$ , please refer to figure (2). It can be seen both from equation (30) and figure (2) that  $z_0$  can be used to control  $\rho$ . In a similar way  $z_0$  can be used to control torque as can be seen in equation (32). The relationship of Torque with  $r$  and  $q$  is straightforward. It can be seen from equation (32) that the torque is directly proportional to  $q$  and inversely proportional to  $r^2$ . Figure (3) also shows that  $q$  can be used to control the torque if desired. It can also be seen from figure (3) which is given for fixed values of  $q, u$  and  $r$  that torque has a maximum value of the order  $10^{-13}$  for each value of inclination  $i$ .

## 5.2 Equilibrium 2

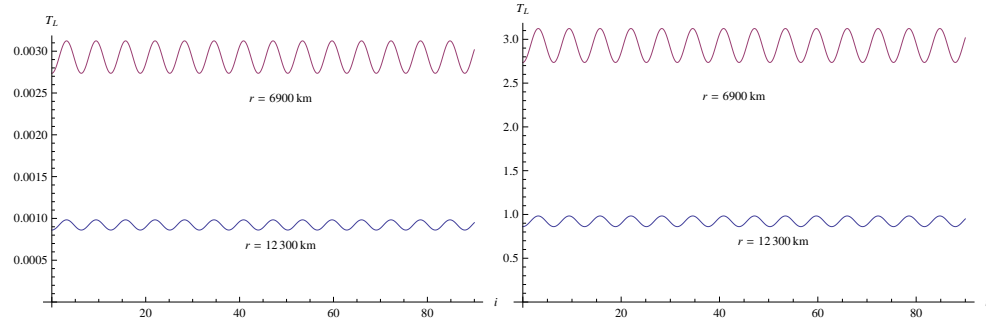
In this equilibrium position the attitude motion of satellite is in the roll direction only. In this case  $\|\rho_0(z_0)\|$  is a linear function of  $z_0$  only. It has a value of  $2.47z_0$ . Torque is a function of the inclination  $i$ , charge  $q$  and  $r$  only.

$$\|T_L(q, i, r)\| = 1.27 \times 10^{16} \frac{z_0}{r^2} |q(0.0011 - 0.0000727 \cos i)| \quad (33)$$

In the same way as in equilibrium one, it is directly proportional to  $q$  and inversely proportional to  $r^2$ . Unlike equilibrium one, Torque in this case is a periodic function of the inclination  $i$  for fixed values of  $q$  and  $r^2$ . For fixed values of charge  $q = 0.01C$ , or  $q = 10C$ ,  $z_0 = 1$ , and  $r = 6900km$  or  $r = 12300$  the optimum values of torque changes periodically. To see the periodic behavior of the torque and a comparison of the torque for two different values of  $r$ , see figure (4). From the comparison for  $r = 6900km$  and  $r = 12300km$  we can see that the value of the Lorentz torque is higher in Low Earth Orbits (LEO). When charge is increased from  $0.01C$  to  $10C$  the magnitude of Lorentz torque increase



**Fig. 3**  $\|T_L(q, u, i, r)\|$  for fixed values of charge  $q = 10000C$ (left),  $100C$ (right),  $r = 6900km$  and  $u = 40$  in the case of equilibrium 1



**Fig. 4**  $\|T_L(q, u, i, r)\|$  for fixed value of charge  $q = 0.01$  (left),  $q = 10$  (right) and  $z_0 = 1$  in the case of equilibrium 2

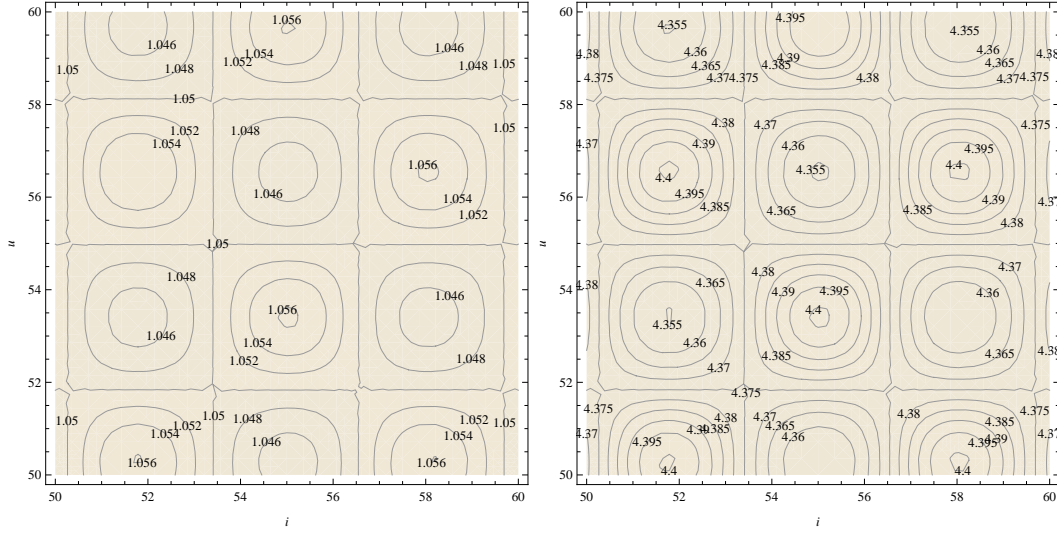
significantly. It means electrostatic charge can be used as some type of control if desired. This can be seen in figure (4).

### 5.3 Equilibrium 3

In this case  $\|\rho_0\|$  is a linear function of  $z_0$ . It has a value of  $1.42138z_0$ . Torque in this case is zero. The attitude motion of the satellite is in the pitch direction and the electrostatic of the screen surface is almost constant which makes the components of Lorentz Torque zero.

### 5.4 Equilibrium 4

This position is a special case which can happen only when the orbital system coincides with the principal axis of inertia which is rigidly fixed to the satellite. For equilibrium 4 described in section 4,  $\|\rho_0\|$  and  $\|T_L\|$  are determined in the same way as in the case of equilibrium 1.



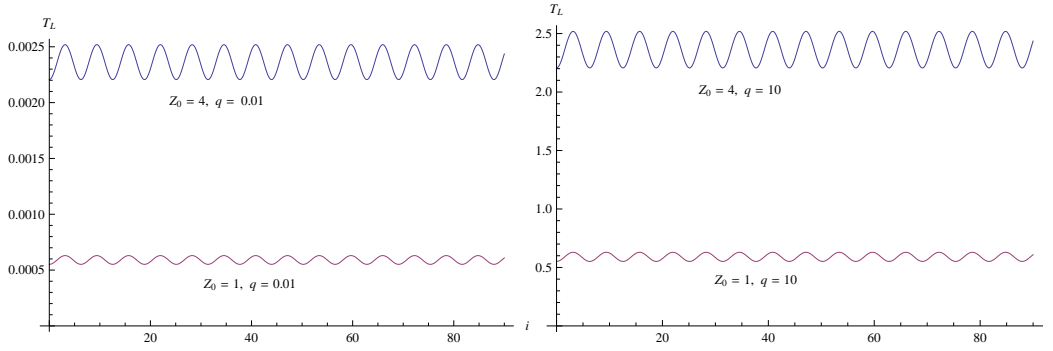
**Fig. 5** a. Contour plot of  $\|\rho_0(u, i)\|$  with maximum and minimum values occurring more than once confirming its periodic behavior.  $z_0$  is taken to be 0.96 ( equilibrium 4) b. Contour plot of  $\|\rho_0(u, i)\|$  with maximum and minimum values occurring more than once confirming its periodic behavior.  $z_0$  is taken to be 4 ( equilibrium 4).

$$\|\rho_0(u, i, z_0)\| = z_0 \sqrt{1 + \frac{2.98(0.11 - 0.727 \cos i)^2}{D_{eq4}} + \frac{1.57(\cos u \sin i)^2}{D_{eq4}}}, \quad (34)$$

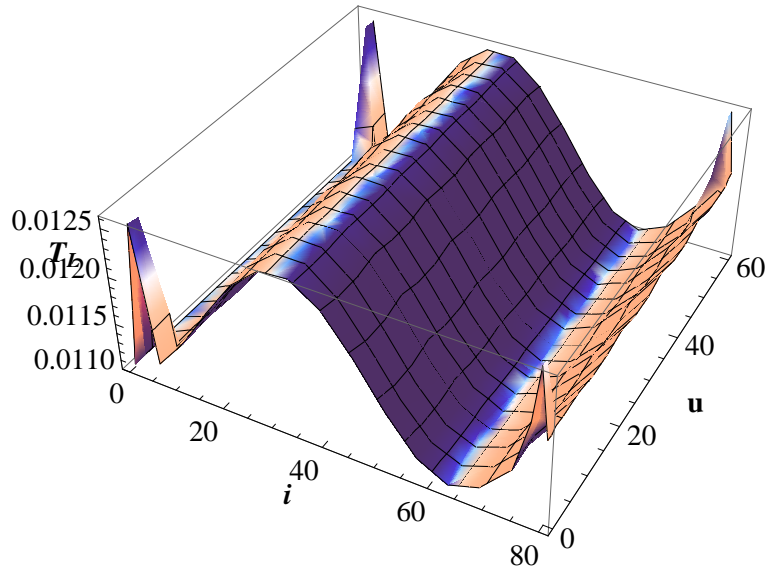
$$D_{eq4} = (42.8 - 2.83 \cos i - 1.37 \cos u \sin i)^2.$$

$$\|T_L(q, u, i, z_0, r)\| = \frac{q z_0 \times 10^{11}}{r^2} \sqrt{\frac{(19 - 1.25(\cos i + \cos u \sin i))^2 + (19 - 1.25(\cos i - \cos u \sin i))^2}{+ (3.6 - 47.6 \cos i + 1.57 \cos^2 i + 1.57 \cos^2 u \sin^2 i)^2} \frac{1}{D_{eq4}}}. \quad (35)$$

It can be seen from equations (34) that  $\|\rho_0(u, i, z_0)\|$  is independent of  $r$  even though its components depend on it. Equation (35) gives the magnitude of torque.  $\|\rho_0(u, i, z_0)\|$  is an almost periodic function of inclination  $i$  and latitude  $u$  with a maximum value of 1.057 meters and minimum value of 1.04611 meters for  $z_0 = 0.96$ . As the function is almost periodic therefore these optimum values occur at various values of  $i$  and  $u$ . For example the maximum occurs at  $(i, u) = (61.33, 34.56)$ . Similarly the minimum occurs at  $(i, u) = (58.05, 53.40)$ . For some other occurrences of the optimum values, see figures (5). It can be seen both from equation (34) and figure (5) that  $z_0$  can be used to control  $\rho_0$ . In a similar way  $z_0$  can be used to control torque as can be seen in equation (35). The relationship of Torque with  $r$  and  $q$  is straightforward. It can be seen from equation (35) that the torque for equilibrium four is directly proportional to  $q$  and inversely proportional to  $r^2$ . Therefore  $q$  and  $z_0$ , can be used to control torque if desired. To completely describe the torque, its representative graph is given in figures (6). In the same way as in equilibrium 2 when charge is increased from  $0.01C$  to  $10C$  the magnitude of Lorentz torque increases significantly. It means electrostatic charge can be used as some type of control if desired which can be seen in figures (6). It can also be seen from figure (7) which is given for fixed values of  $q = 0.01C$ ,  $z_0 = 2$  and  $r = 6900km$  that torque has a maximum value of the order  $10^{-2}$  for each value of inclination  $i$ .



**Fig. 6**  $\|T_L(q, u, i, r)\|$  for fixed value of altitude ( $r = 6900km$ , latitude ( $u = 20$ ) and two different values of  $z_0 = 1, z_0 = 4$ , and  $q = 0.01C, q = 10C$  in the case of equilibrium 4



**Fig. 7**  $\|T_L(q, u, i, r)\|$  for fixed values of  $q = 0.01$ ,  $z_0 = 2$  and  $r = 6900km$  in the case of equilibrium 4

## 6 CONCLUSIONS

To control the attitude of a general shape charged satellite we proposed the utilization of a Lorentz torque with the gravity gradient torque. The effect of Lorentz torque on the attitude dynamics and the orientation of the equilibrium positions is discussed. The satellite is assumed to move in a circular orbit in the geomagnetic field. For this particular setup we derived four equilibrium positions. The attitude motion for these equilibrium positions is analyzed in detail for different values of charge ( $q$ ), charged center of the satellite relative to its center of mass ( $\rho_0$ ), inclination, and latitude. The numerical results confirm that the Lorentz torque has a significant effect on the attitude orientation of satellite for any inclination, specially in highly inclined orbits.

In the case of equilibrium 1, 2 and 4, it is shown that the value of charge  $q$  can control the magnitude of the Lorentz torque. We can choose the optimal torque to create natural force which can be used to control the attitude of the satellite. In case of equilibrium 1, a very high amount of charge is needed to generate a reasonable amount of torque. That is, a  $1000C$  charge is needed to generate Lorentz torque of the order  $10^{-13}$ . On the other hand, in case of equilibrium 2 and 4 a charge of  $0.01C$  will generate a torque of the order  $10^{-3}$ . This means that the use of charge as a control is a more realistic option in equilibrium 2 and equilibrium 4. This also means that, Lorentz force can be used to control satellite without consuming too much propellant. The installation of such control on a satellite is dependent on the size of the surfaces of the satellite, and the screen charging, which can be realized by manufacturing a system of electrodes simulating the controlled electrostatic layer. Such kind of control may be used instead of the magnetic control system, as it is easy to control the mass of the satellite and decrease the cost.

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